Abstract

As a validation of the new specific finite element model for cables described in Part I, numerical tests and comparisons between numerical, theoretical and experimental results are presented in this paper. The model proves to be accurate and economical in terms of degrees of freedom required, compared to standard models using 3D elements. Moreover, the model makes it possible to quantify the influence of the interwire contact conditions on the response of the cable. For a cable made of one core and one layer of helical wires, it appears that the interwire pivoting drives the cable response in axial loading. Moreover, it is shown that this pivoting can be considered as frictionless. By contrast, it is the interwire sliding which controls the cable response in bending. Through an application in circular bending, the model proves to be reliable and precise.

1 Introduction

In the first part of this study, the theoretical bases of a new specific finite element model for cables have been presented. The finite elements introduced to represent the wires of a cable are curved beam elements with 4 nodes and 6 degrees of freedom per node and are based on a Cartesian isoparametric formulation. In combination with realistic interwire contact conditions enforced using the Lagrange multipliers method, the FE model makes it possible to study every possible interwire motions that can occur in a cable. We have determined that the combinations of sliding (S), rolling (R) and pivoting (P) give the 8 contact cases listed in Table 1.

The goal of this second part is to validate the FE model through numerical tests and comparisons between numerical, theoretical and experimental results available in the literature. First, a numerical study of one helical wire is made, in order to compare the results given using the new curved beam finite element with results from a standard finite element code. Then, the axial loading of a single strand cable will be addressed and the analysis of the influence of the interwire motions on the overall behavior of the cable will be performed. A similar study of the interwire motions will also be carried out for the bending of a single strand cable. Different examples of applications will also be proposed throughout the paper.

2 Numerical study of one helical wire

As a first validation test of the finite element model for cables introduced in Part I, it is interesting to investigate the
accuracy of the new curved beam elements which are used to
discretize the cable wires.

For this purpose, we modeled one pitch length of a steel
helical wire using our 4-node finite element
\( (E = 2 \times 10^{11} \text{ Pa}, v = 0.3) \). The results were compared to
those given by a reference model built with the FE code
SAMCEF\[^{4}\] for different types of loadings, end conditions
and geometries. The characteristics of each model are given
in Table 2 and the results are presented in Table 3. All the
tests were performed with one end clamped and the other
submitted to a force \( F_z \) or \( F_x \) (See Fig. 1 for axes). The
displacements in meters shown in Table 3 are those of the
free end of the wire. Only the translations \( u_x, u_y, u_z \) are
compared since the rotations are not accessible with the
finite elements used in SAMCEF. The relative differences
\( e_{u_x}, e_{u_y}, e_{u_z} \) are calculated with respect to the SAMCEF
model. For instance:

\[
e_{u_x} = \frac{|u_{u_x}^{\text{SAMCEF}} - u_{u_x}^{\text{FE}}|}{u_{u_x}^{\text{SAMCEF}}} \times 100.
\]

The comparative tests show that the discrepancies do not
exceed 1.5\% in all the cases studied. Therefore, the present
FE model is proved to be accurate and economical in terms
of degrees of freedom required.

3 Study of the axial loading of a strand

3.1 Role of the interwire motions

For a whole cable, we can now proceed to a series of
numerical tests aimed at identifying the contact conditions
(presence or absence of sliding, rolling and pivoting) which
drive the axial behavior of a single strand cable.

An axial force is applied to the cable whose ends are either
restrained against rotation or free to rotate. The cable is
discretized over one pitch with 6 of our 4-node elements for
each wire and the core. In the applications below, the cable
is made of six helical wires around a core and the material
and geometric parameters are:

\[
E = 2 \times 10^{11}, v = 0.3, \\
R_z = 2.675 \text{ mm}, R_w = 2.59 \text{ mm}, \alpha = 8.18^\circ.
\]

We will compare the coefficients \( F_x, F_z, M_x \) and \( M_z \)
calculated for each contact case (See Table 1) in the
following load-deformation equation:

\[
\begin{bmatrix}
F_z \\
M_z
\end{bmatrix} =
\begin{bmatrix}
F_x & F_z \\
M_x & M_z
\end{bmatrix}
\begin{bmatrix}
\varepsilon_z \\
\tau_z
\end{bmatrix}
\]

where \( F_z \) is the axial force, \( M_z \) is the twisting moment
acting on the cable, \( \varepsilon_z \) is the cable axial strain and \( \tau_z \) is the
cable angle of twist per unit length. The coefficients \( F_z \) and
\( M_z \) describe the traction-torsion coupling that occurs in a
cable. The numerical values of \( F_x, F_z, M_x \) and \( M_z \) are
given in Table 4. The first noticeable thing is that, contrary
to most of the mathematical cable models (Cardou and
Jolicoeur, 1997), the present FE model provides a symmetric
matrix of axial behavior. Secondly, it appears that the
contact cases can be gathered in 2 groups of 4: group A
(cases 1, 3, 4, 7) includes the cases where pivoting is
prevented and group B (cases 2, 5, 6, 8) includes the cases
where pivoting is free. Indeed, in group B, the relative
differences between the coefficients do not exceed 0.5\% and
in group A, the differences are negligible. From these
results, it can be concluded that rolling and sliding play no
significant role in the axial response of a single strand cable.

Rolling+Sliding (R+S)
Rolling+Sliding+Pivoting (R+S+P).

Rolling and sliding may be either present or absent, their
influences on the experimental tests are likely to be
undetectable. Otherwise, it is worth mentioning that, as
expected, no separation occurs between the outside wires
and the core in traction. Separation could occur if the cable
was loaded in compression (Prakash et al., 1992).

3.2 Comparisons with theoretical and experimental data

The results given by the FE model in the two contact cases
determined above have to be compared with experimental
data in order to establish whether pivoting occurs in a real
cable. In the present paper, the experimental work presented
in Utting and Jones (1987) is taken as a reference. We also
display the predictions from the linear theory of wire ropes

In order to compare all the results from the different
sources, we have plotted in Figs. 2, 3, 4 and 5 the
characteristic ratios \( \Delta M_z/\Delta \varepsilon_z \) and \( \Delta M_z/\Delta \tau_z \)
introduced by Jolicoeur and Cardou (1991) and calculated for various
values of the cable lay angle \( \alpha \). These ratios represent the
results of two successive tests in which the cable is subjected
to a constant axial force \( F_z \) while a first value \( M_z^1 \) of the
twisting moment and then a second value \( M_z^2 \) are enforced.
Hence two sets of deformations \( (\varepsilon_z^1, \tau_z^1) \) and \( (\varepsilon_z^2, \tau_z^2) \) are

\[^4\] SAMCEF is a registered trademark of SAMTECH S.A.,
Liege, Belgium.
related to \( \Delta M_z = M_z^2 - M_z^1, \) \( \Delta \varepsilon_z = \varepsilon_z^2 - \varepsilon_z^1 \) and \( \Delta \tau_z = \tau_z^2 - \tau_z^1. \)

Figures 2 and 3 show the results obtained with the present FE model in the free pivoting case as well as the results given by Utting and Jones (1987) and those given by Costello's model (Costello, 1990). Figures 4 and 5 present the characteristic ratios computed using our FE model when pivoting is prevented. From Figs. 2 and 3, it appears that the values computed in free pivoting are very close to the experimental results and the values given by Costello's theory. For the ratio \( \frac{\Delta M_z}{\Delta \varepsilon_z} \) the average relative discrepancy is 3% between the FE model and the experimental results, whereas it reaches 8% between Costello's model and the experimental data. Since the last experimental point in the plot of ratio \( \frac{\Delta M_z}{\Delta \tau_z} \) is probably erroneous, it would be meaningless to discuss the differences with the results given by the models. The values shown in Figs. 4 and 5 when pivoting is prevented are about 10 times greater than those obtained with free pivoting and they are very far from the experimental data. Then, it seems that the FE model with no pivoting has no physical significance and has to be rejected.

The conclusion of this study is that in a real cable, the interwire pivoting is best modeled when considered as free (frictionless). Moreover, the FE model is proved to be capable of giving an accurate prediction of the cable response under axial loading.

4 Study of the bending of a strand

4.1 Role of the interwire motions

As in axial loading, numerical tests have been made in order to explore the role of the different interwire motions when a strand is submitted to small bending. In the previous section, it was shown that the interwire pivoting can be considered as free in axial loading. It means that the friction coefficient for the interwire pivoting is nearly zero and in fact, this result does not depend on the loading. Then, from the 8 possible contact cases recalled in Table 1, only the 4 contact cases with pivoting are studied here. The characteristics of the cable studied are listed in Section 3.1 and each wire is discretized with 6 FE over one pitch. In all that follows, the strand is axially loaded with a force of 10 kN which is sufficient to prevent the wires from separating but is not high enough to create a coupling between the bending and axial behaviors.

The cable ends are restrained against axial rotation and a lateral load per unit length of 5 kN/m is applied to the core. The length of the cable is noted \( L. \) We measure the cable transverse displacement \( u_T (L / 2) \) at mid length, for all the contact cases. The results obtained for one pitch length of the strand are given in Table 5. We can see that sliding has a prominent role on the cable response, and accordingly, the bending stiffness is much larger (about 3 times) when sliding is not allowed than when it is free. Due to friction, the bending stiffness of a real strand would be at an intermediate level between the 2 limiting cases. As in axial loading, rolling has no noticeable influence on the cable response. The same observations have been made for other types of bending loads and different values for the cable length.

4.2 Circular bending of a strand

Since it seems that no experimental study has been made so far, we now address the problem of the circular bending of a strand because it has already been studied analytically by Costello (Costello, 1990). One pitch of the single strand cable described in Section 3.1 is simply supported at both extremities and a constant curvature \( \rho = 2.5 \text{ m} \) is imposed (See Fig. 6). Each wire is discretized using 6 FE and the interwire sliding is considered as frictionless. One can compute the global bending moment at both extremities of the cable, as well as the bending moments in each wire. The results obtained here and those given by Costello's model are shown in Table 6, where the numbering of the wires is made according to Fig. 7.

The conclusion that can be drawn from the analysis of Table 6 is that the present results are satisfactory, because the discrepancies with Costello's model do not exceed 2.2%. Moreover, it is interesting to remark that the distributions of bending moments among the wires are different in the 2 models. As opposed to the present approach, Costello's consists in calculating the total bending stiffness of the cable as the sum of the individual bending stiffnesses of the wires, without taking the interwire contacts into account. Then, all the bending moments in the helical wires are the same using Costello's model, whereas they vary in a given section and along the cable when calculated from the FE model, owing to the helical nature of the wires. The latter model gives a more precise account of the distribution of bending moments in the cable.

The FE model also gives an access to the value of the interwire motions in the cable. Since we have pointed out its strong influence on the cable response in Section 4.1, Fig. 8 shows the interwire tangential sliding between the helical wires and the core (the binormal component of sliding is negligible) along 2 pitches of the strand. We can first state that the interwire sliding is pitch-periodic. In addition, the behaviors of the wires 1 and 4, 2 and 5, 3 and 6 are symmetric with respect to the core. The sliding of the wire 6 on the core is the same as that of the wire 2 with a 1/3-pitch offset. The same remark holds for wires 5 and 3. It is also worth noting that the maximum slidings are reached for the wires 1 and 4 which are located in the plane orthogonal to the plane of the bending (this would be the neutral plane for a straight beam). These findings agree with the predictions made by Lutchansky (1969) in his model of a submarine cable bent over a drum.
5. Conclusion

For the same accuracy, and with more information, the model proves to be economical in terms of degrees of freedom required, compared to standard models using 3D elements. Moreover, it compares favorably with experimental data reported in the literature. In the case of a single strand cable, the model shows that it is the interwire pivoting which drives the axial cable response whereas it is the interwire sliding which controls the cable response in bending. From comparisons with experimental results, the pivoting can be considered as free in a real strand. Through an application in circular bending, we show the capability of the model to precisely describe the local and global behavior of a cable.

Acknowledgements- The authors gratefully acknowledge the support of the National Science Foundation through grant CMS-9502123 and the support of Ecole Centrale de Nantes, France, where the first two authors started their research.

References

Figure 1: Centerline of a helical wire
Figure 2: Slope $\frac{\Delta M_z}{\Delta \varepsilon_z}$ versus $\alpha$ - FE model with free pivoting

Figure 3: Slope $\frac{\Delta M_z}{\Delta \tau_z}$ versus $\alpha$ - FE model with free pivoting
Figure 4: Slope $\frac{\Delta M_z}{\Delta \varepsilon_z}$ versus $\alpha$ - FE model with pivoting prevented

Figure 5: Slope $\frac{\Delta M_z}{\Delta \tau_z}$ versus $\alpha$ - FE model with pivoting prevented
Figure 6: Circular bending of a strand

Figure 7: Wire numbering at z=0 and z=L
Figure 8: Interwire tangential sliding
### Table 1: Intercase contact cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Motions allowed</th>
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<tbody>
<tr>
<td>1</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>R</td>
</tr>
<tr>
<td>4</td>
<td>S</td>
</tr>
<tr>
<td>5</td>
<td>R+P</td>
</tr>
<tr>
<td>6</td>
<td>S+P</td>
</tr>
<tr>
<td>7</td>
<td>S+R</td>
</tr>
<tr>
<td>8</td>
<td>S+R+P</td>
</tr>
</tbody>
</table>

### Table 2: Characteristics of the 2 models of helical wire

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<th>FE type</th>
<th>SAMCEF model</th>
<th>Present FE model</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Hexahedral FE</td>
<td>4-node curved beam FE</td>
</tr>
<tr>
<td>Number of FE (per pitch)</td>
<td>32 nodes - 3 dofs per node</td>
<td>6 dofs per node</td>
</tr>
<tr>
<td>Total number of dofs</td>
<td>3,210</td>
<td>114</td>
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### Table 3: Comparison between SAMCEF and present models

<table>
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<tr>
<th></th>
<th>(R_c = 10^{-3}) m</th>
<th>(R_w = 10^{-3}) m</th>
<th>(\alpha = 8^\circ)</th>
<th>(F_z = 30) N</th>
<th>(u_x^{\text{SAMCEF}})</th>
<th>(u_y^{\text{SAMCEF}})</th>
<th>(u_z^{\text{SAMCEF}})</th>
<th>(u_x^{\text{FE}})</th>
<th>(u_y^{\text{FE}})</th>
<th>(u_z^{\text{FE}})</th>
<th>(e_{u_x})</th>
<th>(e_{u_y})</th>
<th>(e_{u_z})</th>
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<tbody>
<tr>
<td>(R_c = 10^{-3}) m</td>
<td>(R_w = 10^{-3}) m</td>
<td>(R_c = 10^{-3}) m</td>
<td>(R_w = 10^{-3}) m</td>
<td>(R_c = 10^{-3}) m</td>
<td>(R_w = 10^{-3}) m</td>
<td>(R_c = 10^{-3}) m</td>
<td>(R_w = 10^{-3}) m</td>
<td>(\alpha = 8^\circ)</td>
<td>(\alpha = 25^\circ)</td>
<td>(\alpha = 60^\circ)</td>
<td>(\alpha = 8^\circ)</td>
<td>(\alpha = 8^\circ)</td>
<td>(\alpha = 8^\circ)</td>
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<tr>
<td>(u_x^{\text{SAMCEF}})</td>
<td>(-1.54 \times 10^{-3})</td>
<td>(-1.57 \times 10^{-4})</td>
<td>(-2.21 \times 10^{-3})</td>
<td>(+1.53 \times 10^{-3})</td>
<td>(+4.98 \times 10^{-4})</td>
<td>(+5.37 \times 10^{-3})</td>
<td>(+8.80 \times 10^{-6})</td>
<td>(+1.14 \times 10^{-6})</td>
<td>(+1.42 \times 10^{-7})</td>
<td>(+4.84 \times 10^{-5})</td>
<td>(+2.55 \times 10^{-5})</td>
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<tr>
<td>(u_y^{\text{SAMCEF}})</td>
<td>(+4.98 \times 10^{-3})</td>
<td>(+5.37 \times 10^{-3})</td>
<td>(+8.80 \times 10^{-6})</td>
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<tr>
<td>(u_z^{\text{SAMCEF}})</td>
<td>(+1.42 \times 10^{-3})</td>
<td>(+4.84 \times 10^{-5})</td>
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<td>(+4.84 \times 10^{-5})</td>
<td>(+2.55 \times 10^{-5})</td>
<td>(-5.16 \times 10^{-5})</td>
<td>(+1.43 \times 10^{-4})</td>
<td>(+4.85 \times 10^{-3})</td>
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<td>(+4.97 \times 10^{-4})</td>
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<tr>
<td>(u_y^{\text{FE}})</td>
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<td>0%</td>
<td>1.3%</td>
<td>0.6%</td>
<td>0.4%</td>
<td>0.4%</td>
<td>1.4%</td>
<td>0.8%</td>
<td>0.6%</td>
<td>0.2%</td>
<td>0.1%</td>
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Table 4: Numerical results for the 8 contact cases in axial loading

<table>
<thead>
<tr>
<th>Case</th>
<th>Interwire motions allowed</th>
<th>$F_\varepsilon$ (N)</th>
<th>$F_\tau$ (Nm/rad)</th>
<th>$M_\varepsilon$ (Nm)</th>
<th>$M_\tau$ (Nm²/rad)</th>
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<tbody>
<tr>
<td>1</td>
<td>None</td>
<td>0.292 x 10⁸</td>
<td>0.114 x 10⁵</td>
<td>0.114 x 10⁵</td>
<td>0.314 x 10³</td>
</tr>
<tr>
<td>3</td>
<td>R</td>
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<td>0.114 x 10⁵</td>
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<td>0.314 x 10³</td>
</tr>
<tr>
<td>4</td>
<td>S</td>
<td>0.292 x 10⁸</td>
<td>0.114 x 10⁵</td>
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<tr>
<td>7</td>
<td>R+S</td>
<td>0.292 x 10⁸</td>
<td>0.114 x 10⁵</td>
<td>0.114 x 10⁵</td>
<td>0.314 x 10³</td>
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<td>2</td>
<td>P</td>
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<td>0.185 x 10⁵</td>
<td>0.185 x 10⁵</td>
<td>0.553 x 10²</td>
</tr>
<tr>
<td>5</td>
<td>R+P</td>
<td>0.290 x 10⁸</td>
<td>0.185 x 10⁵</td>
<td>0.185 x 10⁵</td>
<td>0.554 x 10²</td>
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<td>6</td>
<td>S+P</td>
<td>0.290 x 10⁸</td>
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<td>0.185 x 10⁵</td>
<td>0.552 x 10²</td>
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<tr>
<td>8</td>
<td>R+S+P</td>
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<td>0.185 x 10⁵</td>
<td>0.185 x 10⁵</td>
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Table 5: Numerical results in bending

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<th>$u_r(L/2)$ (m)</th>
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<tr>
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<td>5</td>
<td>R+P</td>
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<tr>
<td>6</td>
<td>S+P</td>
<td>0.306 x 10⁻³</td>
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<tr>
<td>8</td>
<td>R+S+P</td>
<td>0.307 x 10⁻³</td>
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Table 6: Bending moments (Nm) in the wires

<table>
<thead>
<tr>
<th></th>
<th>$M_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$m_4$</th>
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<th>$m_6$</th>
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<td>Costello</td>
<td>19.958</td>
<td>2.790</td>
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<td>2.790</td>
<td>2.790</td>
<td>2.790</td>
<td>3.217</td>
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<td>FE z=0</td>
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<td>2.811</td>
<td>2.809</td>
<td>2.717</td>
<td>2.811</td>
<td>2.809</td>
<td>2.717</td>
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<tr>
<td>FE z=L</td>
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<td>2.811</td>
<td>2.717</td>
<td>2.809</td>
<td>2.811</td>
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