FRICTIONAL DISSIPATION IN AXIALLY LOADED SIMPLE STRAIGHT STRANDS

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ABSTRACT: In this paper, an analytical investigation is made of the frictional damping properties of axially loaded metallic cables made from one layer of wires helically wrapped around a central wire. Our efforts are focused on the quantity of energy dissipated through friction due to the motions between wires when a cable is loaded. Although the local interwire pivoting drives the response of the cables studied, a first linear model is built where pivoting is allowed, but friction is not taken into account. Then, a law of friction is established and linearized to extend the linear model into a tractable piecewise linear hysteretic one. Through a variety of examples, it appears that the energy dissipated in friction over a load cycle is very small compared to other sources of dissipation, because axially loaded simple straight strands do not experience fretting-induced failures, except close to terminations. It is also shown that modifying the design of such cables is not expected to significantly improve their damping properties.

INTRODUCTION

Thanks to their large axial strength allied with their flexibility, cables are used in many applications ranging from cable-stayed bridges and large-span roofs to tension-leg-platforms and prestressed concrete structures. In service life, these structures do not experience only static loads, but also cyclic efforts such as vortex-induced vibrations or traffic loads. Then, when the damping properties of a cable are mainly due to friction between its constituent wires, these properties can also be expected to be related in some way to the cable durability, because the interwire wear and fretting phenomena involved in damping can drastically shorten the cable fatigue life. In an attempt to quantify the energy dissipated through friction and correlate it with some basic fatigue data, the frictional damping properties of axially loaded metallic cables are explored in this paper.

Although the design of cables is in constant evolution, the study of cable damping properties is still a current research field, even though it was undertaken some decades ago (Pipes 1936; Kawashima and Kimura 1952). A review of experimental work on cable damping in transverse vibrations were made recently (Fang and Lyons 1996). As for the axial damping of cables, Hobbs and Raoof (1984) presented a report of the few available experimental results.

The cable damping properties due to friction are obviously governed by the geometry and the characteristics of the interwire contacts. Hobbs and Raoof (1984) proposed a model for the axial frictional energy dissipation in multilayered spiral strands. Since then, they have also achieved some fatigue predictions (Raoof 1991), but their cable model is based on the homogenization of the cable layers into orthotropic cylindrical sheets and cannot be used for all types of cables.

As a first step, the present investigation is limited to axially loaded simple straight strands made from one layer of wires wrapped around a core. If a simple straight strand has no core, or if the core radius is too small, the outside helical wires are in contact with each other along helical lines. Then, everywhere the static frictional force is overcome, interwire sliding is induced by the change in the lay angle or the unwinding of the cable created by the axial loading. These phenomena have been modeled elsewhere (Huang 1978; Ramsey 1990), and Blakeborough and Cullimore (1984) showed that they result in hysteresis for cyclic axial loads.

Another case of interwire contact is found in current simple straight strands used in prestressed structures and parallel wire bundles. Their design is such that the outside wires are only in contact with the core. Note that the contact is permanent for usual tensile loads. Many models, such as the theory of wire ropes (Costello 1990), are available for this type of cable but the friction effects are generally neglected. More refined models often postulate that interwire sliding might occur, but Utting and Jones (1987) showed that this hypothesis does not hold. Their results indicate that, except possibly near the termination of the strand, there is no slip between the outside wires and the core under axial loads. This has been corroborated through the finite-element model made by Nawrocki (1997). Indeed, it can be illustrated (Nawrocki and Labrosse 1997) that when axial loading is considered, it is the local pivoting of the outside wires on the core that governs the overall cable response. All other possible interwire motions consistent with keeping the wires in contact are negligible and have practically no influence on the cable elastic behavior. Accordingly, they will not be considered here.

In their recent papers, Huang and Vinogradov (1996a,b) investigated the effects of friction forces and interwire slip on the mechanical properties of axially loaded simple straight strands. They also presented an expression of the frictional energy dissipation over one axial load cycle. In their model, it is assumed that slip does occur and that the line interwire efforts are known and constant. The present work aims at removing these hypotheses for the study of the frictional energy dissipation in axially loaded simple straight cables.

The paper is organized as follows. First, a linear model of a simple straight strand with free local interwire pivoting is proposed, which is based on the study of each constituent wire and a geometric compatibility equation. Then, a specific law of friction for pivoting is introduced, and Kirchhoff-Love’s equations of curved rods are used to derive the interwire efforts. Combining the different equations, a hysteretic model of the axial response of a simple straight strand under cyclic loads is proposed. Finally, computed values of frictional damping and experimental values of cable damping are compared to evaluate the importance of friction in pivoting relative to the other sources of dissipation.
LINEAR MODEL OF SIMPLE STRAIGHT STRAND

Cable Geometry

A simple straight wire rope strand is made from one layer of wires of radius \( R_w \) laid helically with an angle \( \alpha \) over a central straight cylindrical core of radius \( R_c \) (Fig. 1). Let \( G^w \) be the center of the wire cross section \( S^w \). The coordinates of \( G^w \) in the reference frame \((O, X, Y, Z)\) are

\[
x = R_c \cos \theta; \quad y = R_c \sin \theta; \quad z = \frac{R_w}{\tan \alpha} \theta (1a-c)
\]

where \( R_w = R_c + R_n \) = helix radius; and \( \theta \) = polar coordinate related to the curvilinear abscissa \( s \) of \( G^w \) defined by

\[
s = \frac{R_w}{\tan \alpha} \theta (2)
\]

Hypotheses

The design of the cable is such that the outer wires are in contact with the core but not with their neighbors. According to Huang (1978), any simple straight stand is expected to take this configuration while axially loaded, provided that the central core and the helical wires are made of the same material. The displacements and deformations are assumed to remain small enough to ensure that the linear theory of elasticity is valid. Indeed, Velinsky (1985) showed that appealing to nonlinear theories was not necessary for typical ranges of axial loads. Moreover, the variation of the wire diameter due to Poisson’s effect and the local flattening in the contact areas are not taken into account because Utting and Jones (1987) demonstrated that their influence on the cable response is negligible. In each wire of the cable, plane sections are supposed to remain plane after deformation. Then, classical displacement fields for beams are taken for every outer wire and the core. Finally, the loads are assumed to be applied quasi-statically.

Theoretical Analysis

Let \( u \), and \( \theta \), be, respectively, the displacement and the rotation of the cable along its axis at \( z \). Under axial loads and assuming a uniform extension \((u_{zz} = 0)\), \( u \), and \( \theta \), are common for the outer wires and the core, because the end grips are supposed to prevent any relative displacement or rotation. Along the contact lines between the wires and the core, and the contact lines between the wires and their neighbors, the local rotation \( \theta \), of the outer wires relative to the core, around the local \( n \)-axis shown in Fig. 1. This pivoting will be referred to as local interwire pivoting. Then, the final form of the displacement fields is presented in (3), where \( G \) is the center of the wire section \( S \), and \( P \) is in \( S \). The superscripts \( w \) and \( c \) refer, respectively, to

![FIG. 1. Strand Geometry](image)

a helical wire and to the core. The external product of vectors is noted as \( \times \).

Helical wire

\[
\mathbf{u}_{\pi} = \mathbf{u}_{z} + \Theta^w \times G^w \mathbf{P}^w + \Omega^w \times G^w \mathbf{P}^w
\]

with \( \mathbf{u}_{z} = u_z(z)\mathbf{Z}, \Theta^w = \theta(z)\mathbf{Z}, \Omega = \theta(z)\mathbf{n} \) (3a)

Core

\[
\mathbf{u}_{\pi} = \mathbf{u}_{z} + \Theta^w \times G^w \mathbf{P}^w \quad \text{with} \quad \mathbf{u}_{z} = u_z(z)\mathbf{Z}, \Theta^w = \theta(z)\mathbf{Z} \quad (3b)
\]

Study of Helical Wires

We will show the developments for the helical wires because those relative to the core are straightforward. Let us first introduce the frame \((G^w, t, n, b)\) where \( t, n, \) and \( b \), are, respectively, the tangent normal, and binormal local vectors along the wire centerline (Fig. 1). From the displacement field, we can derive the local components of the linearized Green-Lagrange tensor of deformations at \( P^w(s, \xi, \eta) \) in the frame \((G^w, t, n, b) \quad e

\[
e_{\xi} = \frac{1}{1 - \frac{\xi}{R}} \left( u_{z} \cos \alpha + (R_c - \xi)\theta_{z} \cos \alpha \sin \alpha - \frac{\theta_{z}}{R} \right) (4a)
\]

\[
e_{\eta} = \frac{1}{1 - \frac{\xi}{R}} \left( -\eta \theta_{z} \cos \alpha + \frac{\theta_{z}}{R} \right) (4b)
\]

\[
\gamma_{m} = 2\varepsilon_{m} = \frac{1}{1 - \frac{\xi}{R}} \left( -\eta \theta_{z} \cos \alpha + \frac{\theta_{z}}{R} \right) (4c)
\]

\[
\gamma_{n} = 2\varepsilon_{n} = \frac{1}{1 - \frac{\xi}{R}} \left( u_{z} \cos \alpha \sin \alpha - (R_c - \xi)\theta_{z} \cos \alpha \right) (4d)
\]

where the helix curvature and the helix torsion are, respectively, given by

\[
\frac{1}{R} = \frac{\sin \alpha}{R_w}; \quad \frac{1}{T} = \frac{\cos \alpha \sin \alpha}{R_w} (5a,b)
\]

On the other hand, according to the classical theory of beams, the components of the stress tensor in \((t, n, b)\) are, for each wire

\[
\sigma_{t} = E\varepsilon_{t}; \quad \sigma_{n} = \sigma_{n0} = \sigma_{w0} = 0; \quad \sigma_{b} = G\gamma_{b}; \quad \sigma_{w} = G\gamma_{w} (6a-d)
\]

where \( E \) = Young’s modulus; \( G = E/2(1 + \nu); \) and \( \nu \) = Poisson’s ratio. The resultant forces and moments in each wire are defined as follows:

\[
F_{t}^{w} = \int_{S^{w}} \sigma_{t} dS; \quad F_{n}^{w} = \int_{S^{w}} \sigma_{n} dS; \quad F_{b}^{w} = \int_{S^{w}} \sigma_{b} dS (7a)
\]

\[
M_{t}^{w} = \int_{S^{w}} (\xi \sigma_{n} - \eta \sigma_{w}) dS; \quad M_{n}^{w} = \int_{S^{w}} \eta \sigma_{w} dS; \quad M_{b}^{w} = \int_{S^{w}} -\xi \sigma_{w} dS (7b)
\]

Neglecting the higher order terms of the curvature and torsion, one obtains

\[
F_{t}^{w} = \pi ER_{w} \cos \alpha(u_{z} \cos \alpha + \theta_{z} R_n \sin \alpha) (8a)
\]

\[
F_{n}^{w} = M_{n}^{w} = 0 (8b)
\]
\[ F'_{u} = \pi GR^2 \cos \alpha (u_{z,2} \sin \alpha - \theta_{z,2} R_0 \cos \alpha) + \pi GR^2 \theta_{n} \]  
\[ M'_{z} = \frac{\pi}{2} GR^4 \cos^2 \alpha \cdot \theta_{z,2} \]  
\[ M'_{z} = \frac{\pi}{4} ER^4_0 \cos \alpha \sin \alpha \cdot \theta_{z,2} \]  

For the whole cable consisting of a core and \( m \) helical outer wires, projections between the local and reference frames yield the total axial forces and moments
\[ F'_{z} = \pi ER^2 u_{z,2} + m(F'_{z} \cos \alpha + F'_{u} \sin \alpha) \]  
\[ M'_{z} = \frac{\pi}{2} GR^4 \theta_{z,2} + m(M'_{z} \cos \alpha + M'_{z} \sin \alpha) \]

\[ + R_0 (F'_{z} \sin \alpha - F'_{u} \cos \alpha) \]  

**Linear Model with Free Local Interverwire Pivoting**

From (8) and (9), one can get \( F'_{z} \) and \( M'_{z} \) in terms of \( u_{z,2} \) and \( \theta_{z,2} \). It is clear that the pivoting still has to be determined through an additional equation. In the case of free local pivoting, this relationship can be provided by the geometric compatibility condition relative to the cable uniform extension. Indeed, the axial strain in the core must be equal to the strain component of the helical wire in the cable axial direction. As \( \theta_{z,2} \) can be seen as the small variation of the lay angle of an outside wire, and similarly to what is done in Costello’s theory (1990), it is illustrated in Fig. 2 that we have
\[ \tan(\alpha + \theta_{z,2}) = \frac{R_0 (2\pi + h \theta_{z,2})}{h(1 + u_{z,2})} \]  
with \( h = \frac{2\pi R_0}{\tan \alpha} \)  

where \( h = \) cable pitch length. After linearization, (10) yields
\[ \theta_{z,2} = -\cos \alpha \sin \alpha \cdot u_{z,2} + R_0 \cos^2 \alpha \cdot \theta_{z,2} \]  

Then, we can get the following linear model of a simple straight strand in matrix form
\[ \begin{bmatrix} F'_{z} \\ M'_{z} \end{bmatrix} = \begin{bmatrix} a^* & b^* \\ b^* & c^* \end{bmatrix} \begin{bmatrix} u_{z,2} \\ \theta_{z,2} \end{bmatrix} \]  

where
\[ a^* = \pi ER^2 \cos \alpha \]  
\[ b^* = m\pi ER_0 \cos \alpha \]  
\[ c^* = \frac{\pi}{2} GR^4_0 + \frac{\pi}{4} ER^4_0 \cos \alpha \sin \alpha \]  
\[ + m \frac{\pi}{4} R^4_0 \cos \alpha \cos \alpha (E \sin^2 \alpha + G(1 + \cos^2 \alpha)) \]  

This set of equations provides a cable homogenized constitutive law in axial loading, but is obviously unable to allow for any energy dissipation due to friction. It is the purpose of the following sections to extend the above equations to build a hysteretic model.

**MODELING OF INTERWIRE CONTACT WITH FRICTION**

In a simple straight strand, the contact zone between a helical wire and the core forms a narrow strip whose central line is a helix. However, for the typical helix angles and ratios \( R_1/R_0 \) used in steel cables, the real contact can be locally approximated as the contact between two parallel straight cylinders subjected to a combination of compression, pivoting, and shear (Fig. 3). This analogy makes it possible to derive the analytical expression of the pivoting moment such that sliding can occur throughout the width of the contact strip. It is assumed that the overall cable loading is large enough for all the outside wires to remain in contact with the core. Because the contact width is very small compared with the wire radius and the contact length, the conditions of application of Hertz’ theory are satisfied (Johnson 1985). Then, the half-width of contact \( a \) is given as a function of the material characteristics, the wire radii, and the normal load per unit length \( N \) on the interwire contact centerline as follows:
\[ a = 2 \sqrt{\frac{2(1 - v^2)R_1 R_0 N}{\pi E(R_0 + R_1)}} \]  
and the pressure distribution \( p(\eta') \) over the contact width is semieliptical, with
\[ p(\eta') = \frac{2N}{\pi a} \sqrt{1 - \eta'^2} \]  

where \(-a \leq \eta' \leq a\).

The pivoting of the section of an outside wire on the core induces a sliding motion in the contact strip. At a contact point between the two cylinders, sliding can develop only if, locally, the distribution of tangential pressure \( q(\eta') \) is such that
\[ |q(\eta')| = \mu p(\eta') \]  

where \( \mu = \) Coulomb’s friction coefficient relative to the material of the cylinders. From (13) and (14), one can obtain the moment \( m_{n\text{max}} \) required to ensure that all the contact points can slide while pivoting
\[ m_{n\text{max}} = 2 \int_{0}^{a} \mu p(\eta') \eta' \ d\eta = k^* \cdot N^{\frac{1}{2}} \]  

where \( k^* = (8\mu/3\pi) \sqrt{2(1 - v^2)R_1 R_0 / \pi E(R_0 + R_1)} \). The equation above extends the usual law of friction established by Coulomb to the case of total pivoting with friction.

**HYSTERETIC MODEL OF AXIALLY LOADED SIMPLE STRAIGHT STRAND**

When the local interwire pivoting is supposed to take place with friction, two cases have to be considered depending on whether \( m_{n\text{max}} \) is reached or not. For simplicity reasons, in this paper, pivoting is strictly prevented as long as \( m_n \leq m_{n\text{max}} \).
Combining the friction law with the linear model of a simple straight strand, it is possible to describe the cable response for a cyclic axial loading, provided that \( m_c \) and \( N \) can be calculated in terms of the loading. Contrary to the assumption made in the models that are cited in the Reference Appendix, the interwire normal force \( N \) is not taken to be constant over a loading cycle.

### Equations of Curved Rods

For a single wire, with \( m_c \) and \( N \) now, respectively, defined as the external moment of pivoting per unit length and the external normal effort per unit length along the contact line between a helical wire and the core, Kirchhoff-Love’s equations for curved rods (Love 1952), assuming a uniform extension of the cable, give

\[
m_c = F^c - \frac{M_0^c}{T} + \frac{M_n^c}{T} \quad N = -\frac{F^c}{R} + \frac{F^w}{T}
\]

(17a,b)

where \( F^c, M_0^c, \) and \( M_n^c \) are calculated as functions of the cable loading after (8) and (9).

### Linearized Law of Friction

The presence of the term \( N^{3/2} \) in the friction law makes the model nonlinear. However, numerical tests prove that for a load cycle, the responses computed either with the nonlinear model (Newton-Raphson’s method of resolution) or with a linearized model are quite similar because \( k^* \) is very small for typical steel cables (with wire radii of a few millimeters and \( \mu \in [0.1, 0.6] \), \( k^* \) is on the order of \( 10^{-7} \)). We can therefore take the following linearized law of pivoting friction to make the hysteretic model piecewise linear

If \( |m| < |k| \cdot |N| \), then \( \theta_n \) is set to a constant (i.e., no pivoting);

Else \( m = k \cdot |N| \) and then \( \theta_n \) is a function of \( u_{z-c} \) and \( \theta_{z-c} \)

after (8) and (17)

(18)

In this notation, \( k = k^* \sqrt{|N_m|} \) where \( N_m \) is the value of \( N \) for the maximum load over a cycle.

Everything is now ready for the study of a load cycle, from a minimum load (m) to a maximum load (M), with a pivoting threshold in loading (l) and unloading (u). Over one load cycle (Fig. 4), \( u_{z-c} \) and \( \theta_{z-c} \) will, respectively, start from the values \( u_{z-c} \) and \( \theta_{z-c} \) and \( \theta_{z-c} \), then successively reach the values \( u_{z-c} \) and \( \theta_{z-c} \), \( u_{z-c} \), \( \theta_{z-c} \), and \( \theta_{z-c} \), and \( u_{z-c} \) and \( \theta_{z-c} \) and \( \theta_{z-c} \), and then back to \( u_{z-c} \) and \( \theta_{z-c} \) and \( \theta_{z-c} \). On the other hand, when it is not prevented, the pivoting angle \( \theta_n \) will evolve from \( \theta_{z-c} \) to \( \theta_{z-c} \).

### Four Phases of Load Cycle

#### Phase 1: from (m) to (l)

After (8) and (9), the load-deformation relationship is

\[
\begin{bmatrix} F_c \\ M_c \\ \theta_n \end{bmatrix} = \begin{bmatrix} a & b & d \\ b & c & e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{z-c} \\ \theta_{z-c} \end{bmatrix}
\]

(19)

with

\[
a = \pi ER^2 + m\pi R^2 \cos \alpha(E \cos \alpha + G \sin \alpha) \\
b = m\pi R \cos \alpha \sin \alpha(E - G) \\
c = \frac{1}{2} \pi G R^2 + m\pi R^2 \cos \alpha(E \sin \alpha + G \cos \alpha) \\
+ m \pi R^2 \cos \alpha(E \sin \alpha + G \cos \alpha) \\
d = m\pi GR^2 \sin \alpha \\
e = -m\pi GR^2 \cos \alpha
\]

The interwire pivoting is prevented and the angle \( \theta_n \) is set to \( \theta_{z-c} = 0 \) for the first cycle, or \( \theta_{z-c} \) as in Phase 3 for other cycles, until interwire pivoting is possible. The pivoting threshold (l) is attained when \( m_c = k \cdot |N|, \) i.e., \( F^c \cdot u_{z-c} + g \cdot \theta_{z-c} + \theta_{z-c} = 0, \) where

\[
f = \cos \alpha \sin \alpha (1 + \psi \cos \alpha)
\]

(20a)

\[
g = -R_n \cos \alpha(1 - \psi \sin \alpha \tan \alpha)
\]

(20b)

\[
\psi = \frac{kE \sin \alpha}{G(R_e - k \cos \alpha \sin \alpha)}
\]

(20c)

In these expressions, second order terms are neglected.

#### Phase 2: from (l) to (M)

The pivoting angle \( \theta_n \) develops from \( \theta_{z-c} \) to \( \theta_{z-c} \) with \( F^c \cdot u_{z-c} + g \cdot \theta_{z-c} + \theta_{z-c} = 0, \)

\[
\begin{bmatrix} F_c \\ M_c \\ \theta_n \end{bmatrix} = \begin{bmatrix} a & b & d \\ b & c & e \\ f & g & 1 \end{bmatrix} \begin{bmatrix} u_{z-c} \\ \theta_{z-c} \\ \theta_n \end{bmatrix}
\]

(21)

with \( \theta_{z-c} = f \cdot u_{z-c} + g \cdot \theta_{z-c} + \theta_{n} \)

The variables \( u_{z-c} \) and \( \theta_{z-c} \) noted with the subscript “l” are calculated at the pivoting threshold (l). The constant \( \theta_{z-c} \) is introduced for convenience and consistency of the matrix model, but does not represent the pivoting angle \( \theta_n \). The maximum pivoting angle \( \theta_{z-c} \) reached when the maximum loading (M) is applied, and then Phase 3 starts.

#### Phase 3: from (M) to (u)

As in Phase 1, the load-deformation relationship is

\[
\begin{bmatrix} F_c \\ M_c \\ \theta_n \end{bmatrix} = \begin{bmatrix} a & b & d \\ b & c & e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{z-c} \\ \theta_{z-c} \end{bmatrix}
\]

(22)

As soon as the unloading starts, \( k \) is converted to \( -k \) so that \( m_c \) opposes \( \theta_n \). The interwire pivoting is prevented, and the angle \( \theta_n \) is set to \( \theta_{z-c} \) calculated at (M), until pivoting is possible. The pivoting threshold (u) is attained when \( m_c = -k \cdot |N|, \) i.e., \( F^c \cdot u_{z-c} + g \cdot \theta_{z-c} + \theta_{n} = 0, \)

\[
\tilde{f} = \cos \alpha \sin \alpha (1 + \psi \cos \alpha)
\]

(23a)

\[
\tilde{g} = -R_n \cos \alpha (1 - \psi \sin \alpha \tan \alpha)
\]

(23b)

\[
\tilde{\psi} = \frac{kE \sin \alpha}{G(R_e + k \cos \alpha \sin \alpha)}
\]

(23c)

#### Phase 4: from (u) to (m)

The pivoting angle \( \theta_n \) develops from \( \theta_{z-c} \) to \( \theta_{z-c} \) with \( F^c \cdot u_{z-c} + g \cdot \theta_{z-c} + \theta_{n} - \theta_{z-c} = 0, \) and then

![FIG. 4. Modeling of Cyclic Loading](image)
The variables \( u_{ew} \) and \( \theta_{ew} \) noted with the subscript "u" are calculated at the pivoting threshold (u). The constant \( \theta_{ew} \) is introduced for convenience and consistency of the matrix model, but does not represent the pivoting angle \( \theta_{ew} \). Phase 4 lasts until the maximum loading \( (m) \) is reached and Phase 1 restarts. The angle \( \theta_{ew} \) will be set to \( \theta_{em} \) calculated at \( (m) \), and \( k \) will have its sign changed once again. Then, four matrix relations relative to the four phases of a general cycle, as shown above, describe the hysteretic model.

**Energy Dissipated in Friction over Axial Loading Cycle**

Noting \( \{ \varepsilon \}^T = \{ u_{ew}, \theta_{ew} \}^T \) and \( \{ \sigma \}^T = \{ F, M \}^T \), the energy dissipated in friction per unit length over a closed load cycle is defined by \( E_{\text{friction}} = \int \{ \sigma \}^T \{ d\varepsilon \} \). Then, from (19), (21), (22), and (24),

\[
E_{\text{friction}} = \frac{1}{2} \left[ \{ \varepsilon_m \}^T \begin{bmatrix} df & ef \\ dg & eg \end{bmatrix} \{ \varepsilon_m \} + \{ \varepsilon_m \}^T \begin{bmatrix} df & ef \\ dg & eg \end{bmatrix} \{ \varepsilon_m \} \right]
- \left[ \{ \varepsilon_i \}^T \begin{bmatrix} df & ef \\ dg & eg \end{bmatrix} \{ \varepsilon_i \} - \{ \varepsilon_i \}^T \begin{bmatrix} df & ef \\ dg & eg \end{bmatrix} \{ \varepsilon_i \} \right] (25)
\]

On the other hand, for a unit length, the elastic strain energy input is given by \( E_{\text{strain}} = \int \{ \sigma \}^T \{ d\varepsilon \} \), in which the relation between \( \{ \sigma \} \) and \( \{ \varepsilon \} \) is approximated for convenience by the linear model with free interwire pivoting. Then, after (12)

\[
E_{\text{strain}} = \frac{1}{2} \left[ \{ \varepsilon_m \}^T \begin{bmatrix} a & b \\ b & c \end{bmatrix} \{ \varepsilon_m \} - \{ \varepsilon_m \}^T \begin{bmatrix} a & b \\ b & c \end{bmatrix} \{ \varepsilon_m \} \right] (26)
\]

Calculating the specific loss per cycle

\[
\Psi_{\text{friction}} = \frac{E_{\text{friction}}}{E_{\text{strain}}} \tag{27}
\]

gives a direct insight into the frictional damping of the cable. Indeed, \( \Psi_{\text{friction}} \) is two times the value of the logarithmic decrement of the axial displacement, which is classically used in studies of damping (Hobbs and Raoof 1984). When the cable specific loss is measured, the total energy dissipation includes the material energy \( E_{\text{material}} \) and the frictional energy \( E_{\text{friction}} \). Hence, \( \Psi_{\text{cable}} = \Psi_{\text{material}} + \Psi_{\text{friction}} \), where

\[
\Psi_{\text{material}} = \frac{E_{\text{material}}}{E_{\text{strain}}} \tag{28}
\]

**DISCUSSION OF NUMERICAL RESULTS**

Some numerical examples are now proposed to evaluate the information given by the model. The simple straight strands studied below have the following characteristics: \( R_e = 2.675 \text{ mm} \); \( R_o = 2.590 \text{ mm} \); \( E = 2 \times 10^7 \text{ Pa} \); \( \nu = 0.3 \); \( m = 6 \); and \( \alpha = 8.18^\circ, 11^\circ \), and \( 14^\circ \).

The last two values of \( \alpha \) are taken to investigate the influence of the lay angle on the results but do not correspond to a real cycle. A realistic value of the friction coefficient \( \mu \) is taken to be 0.12, as in Hobbs and Raoof (1984).

Figs. 5 and 6 show \( \Psi_{\text{friction}} \) related to the cyclic load range \( \Delta F/F_{\text{mean}} \) for cables with ends either fixed (Fig. 5) or free to rotate (Fig. 6). The value of \( F_{\text{mean}} \) is 70 or 40 kN (i.e., 42 and 25%, respectively, of the cable related tensile strength). It can be seen that the values of \( \Psi_{\text{friction}} \) are much larger when the cable ends are free to rotate, however, they hardly reach \( 10^{-4} \), which is very small. One can also mention that \( \Psi_{\text{friction}} \) increases noticeably as the lay angle \( \alpha \) becomes greater. Other numerical tests presented by Labrosse (1998) illustrate that \( \Psi_{\text{friction}} \) is linearly dependent on the friction coefficient \( \mu \) and that when the number of helical wires is allowed to change, \( \Psi_{\text{friction}} \) reaches its highest value for \( m = 6 \).

The maximum experimental values of \( \Psi_{\text{cable}} \) for simple straight strands are reported to average a few percent (Kawashima and Kimura 1952). The geometry of the cables studied in Kawashima and Kimura’s paper is not known, but the comments made up by them strongly suggest that the outside wires contacted each other. As a result, in their experiments, gross slip between the outside wires was likely to induce some dissipation due to friction in addition to that due to the material. Then, because the reported values of \( \Psi_{\text{cable}} \) are much larger than the computed values of \( \Psi_{\text{friction}} \), it can be concluded that the energy dissipated in gross pivoting with friction is negligible compared with the viscous and slip frictional dissipation in the cable.

Considering the very small influence of friction for the type of cables considered here, it is logical that Costello’s model (1990), in which pivoting is free, and the present model give nearly identical results. An illustration of this remark is given...
in Fig. 7, which presents the axial cyclic response of a simple straight strand with the same characteristics as those given above \((\alpha = 8.18^\circ)\) from the two models and from the measurements carried out by Le Laboratoire des Ponts et Chaussées (Nantes, France), with \(F_z\) ranging from 0 to 150 kN. The hysteretic loop is so thin that the curves relative to the loading and unloading cannot be distinguished. The maximum relative discrepancy between theory and experiments is 3%.

CONCLUSIONS

In this paper, we introduced a hysteretic model for axially loaded simple straight wire rope, when the outside wires are only in contact with the core. It has been illustrated by a few examples that friction in gross pivoting is a much smaller source of dissipation than plastic flow and gross slip (when the geometry of the cable makes it possible). Therefore, the friction phenomena in pivoting can be neglected and the linear model with free pivoting and the hysteretic model presented in this paper are very close to Costello’s model.

Although friction and thus wear phenomena have been shown to be negligible, the present model can be seen as an account for the experimental fact that axially loaded simple straight wire rope strands seldom fail due to interwire fretting fatigue, except close to the end grips due to the load effects induced by the clamping. Moreover, it has been shown that no significant extra damping can be achieved by modifying the cable geometry or the end conditions.

More generally and for any kind of cable, it is also possible to conclude from this work that whenever interwire sliding and interwire pivoting can take place on an interwire contact line, for instance in a bent cable, the friction phenomena associated with pivoting can be neglected in the study of the cable damping procedures. Indeed, the local pivoting friction conditions for the points on the interwire contact strips are approximately the same whatever the cable loading.

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APPENDIX. REFERENCES


